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A POINT EXPLOSION IN AN INHOMOGENEOUS ATMOSPHERE

- USSR -

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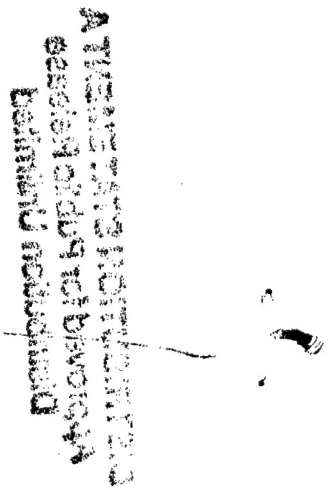

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A POINT EXPLOSION IN AN INHOMOGENEOUS ATMOSPHERE

/Following is a translation of an article by A. S. Kompaneneyets in the Russian-language periodical Doklady Akademii nauk SSSR (Reports of the Academy of Sciences USSR), Moscow, Vol. 130, No. 5, pages 1001-1003./

As is well known, the problem of a point explosion in a medium of constant isentropic index γ has an analytical solution(1) for types of phenomena in which it is possible to neglect the energy in the medium up to the time of the explosion with respect to the energy carried in the detonation wave. A relatively simple solution can be obtained, because the problem can be put into a dimensionless form: it contains no characteristic parameters with the dimensions of length, velocity, or time. If there is a density gradient in the medium, even if only in one direction, it is dimensionless formulation is no longer possible and an exact solution is not obtained. For small gradients the problem can be considered using a pertubation method(2,3).

If the explosion occurs in an extremely rarefied atmosphere, a strong shock wave with a limiting compression is propagated for such a distance that the density is changed by many times with respect to the density at the site of the detonation. In this case it is insufficient to lineraze the equations. The numerical solution of the problem with three variables is very laborious, even using an electronic computer.

A semiqualitative method may be proposed that is based on an essential property of the exact spherically symmetric solution. In this solution the energy is found to be propagated almost uniformly through the whole volume of the detonation wave, and it is only close to the front itself that it exceeds by a factor of 2 or 3 the mean value throughout the volume. The whole mass of the material is also distributed concentrically throughout this region.

It is natural to assume that a detonation wave in an inhomogeneous atmosphere also possesses this property. Actually, if the pressure within the wave is constant in space (the pressure is proportional to the energy density), and the mass density equals zero, the hydrodynamic equations in the main volume are trivially satisfied. Thus, in order to describe the propagation of the wave, it is necessary to use the conditions in the shock front itself.

If the equation of the wave front is $f(r,z,t) = 0$, the normal velocity component of the front D_n is defined by the well known equation,

$$D_n = -\frac{\partial f}{\partial t} \Big/ |\nabla f| = \sqrt{p^2\left(\frac{1}{p} - \frac{1}{p}\right)} \quad (1)$$

Here, as is usual in the consideration of strong waves, the initial pressure is neglected with respect to the pressure of the wave front p. In this approximation the density behind the front p' is connected with the density before the front p by the constant relationship

$$\frac{p'}{p} = \frac{\gamma + 1}{\gamma - 1} \quad (2)$$

The pressure is expressed in terms of the energy density ε by

$$p = (\gamma - 1)\varepsilon = (\gamma - 1)\lambda\frac{E}{V}, \quad (3)$$

where E is the energy density of the detonation; V is the volume occupied by the detonation wave; and $\lambda = \lambda(\gamma)$ is a coefficient indicating how many times larger the energy density near the front is than the mean density throughout the volume (1). The assumption that λ is constant over the surface is the basis of the method here proposed.

We shall consider that the equation for the wave front in cylindrical coordinates is solved for the radius: $r = r(z,t)$. Then the total volume of the wave is

$$V(t) = \pi \int_{z_1}^{z_2} r^2(z, t)\, dz, \quad (4)$$

where $r(z_1, t) = z_2, t) = 0$. Substituting (2), (3), and (4) in (1), and expressing the density by the barometric formula, we obtain a partial differential equation for the function r:

$$\left(\frac{\partial r}{\partial y}\right)^2 - e^{-z/z_0}\left[\left(\frac{\partial r}{\partial z}\right)^2 + 1\right] = 0. \quad (5)$$

Here z_0 is the equivalent thickness of the atmosphere, and y is an auxiliary variable defined by the equation

$$y = \int_0^t \frac{dt}{\sqrt{V}} \sqrt{\frac{\lambda E(\gamma^2 - 1)}{2p_0}}; \quad (6)$$

P_o is the initial air density at the site of the detonation ($z = 0$).

The equation (5) is soluble by the method of separation of variables

$$r = \xi y + \int_0^z dz \sqrt{\xi^2 e^{-z/z_o} - 1}; \qquad (7)$$

$$\frac{\partial r}{\partial \xi} = y + \int_0^z dz \frac{\xi e^{-z/z_o}}{\sqrt{\xi^2 e^{-z/z_o} - 1}} = F(\xi). \qquad (8)$$

For small t or y the wave may be spherical. For this to happen it is sufficient to put the function $F(\xi)$ equal to zero. Then by eliminating ξ from (8) and substituting in (7), we get

$$r = 2z_o \arccos\left[\frac{1}{2} e^{z/2z_o}\left(1 - x^2 + e^{-z/z_o}\right)\right]; \qquad (9)$$

here $x = y/2z_o$.

Thus we obtain the position of the upper and lower points of the wave z_1 and z_2

$$e^{-z_{1,2}/2z_o} = \mp x, \qquad (10)$$

as well as the position and magnitude of its maximum radius

$$e^{-z_m/z_o} = 1 - x^2, \quad r_m = 2z_o \arcsin x. \qquad (11)$$

Thus the maximum possible radius of the wave is equal in this case to $x = 1$, so that the upper limit of thewwave goes out to infinity. But this occurs after a finite time τ which is defined from (6) as

$$\tau = \sqrt{\frac{8m z_o^5 p_o}{\lambda E (\gamma^2 - 1)}} \int_0^1 \sqrt{f(x)}\, dx, \qquad (12)$$

$$\Lambda(x) = \int_{-2\ln(1+x)}^{-2\ln(1-x)} du \arccos^2 \left[\frac{1}{2} e^{u/2} (1 - x^2 + e^{-u}) \right]. \quad (13)$$

The time for the upward wave to go to infinity remains finite, thanks to the fact that the velocity of the wave, according to (1), tends to infinity as $z \longrightarrow \infty$. Fig. 1 shows the curves for the transformation from x to t and $[\Lambda(x)]^{1/2}$. Drawn to scale in Fig. 2 are the calculated cross sections of a wave in a vertical plane passing through the site of the detonation, for various instants of time.

It is obvious that the solution loses its significance before z_1 becomes infinite. Nonetheless it is possible to make the following conclusion: however large the total energy of the detonation, a strong shock wave can be propagated downward, according to the law obtained here, by not more than 1.38 z_0 or about 11 km. With further downward propatation, the shock front will be attenuated more quickly than the following wave on account of the rarefaction waves proceeding upward from it into empty space. The propagation of a wave through unperturbed air will recall the short shock in a substance adjoining a vacuum, which was considered by Ya. B. Zel'dovich (4).

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FIGURE APPENDIX

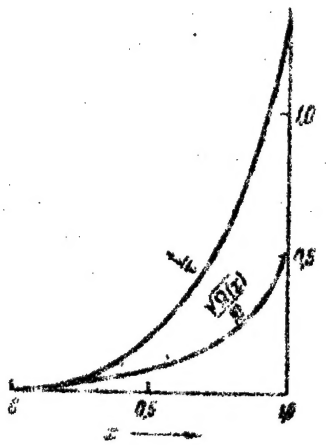

Fig. 1

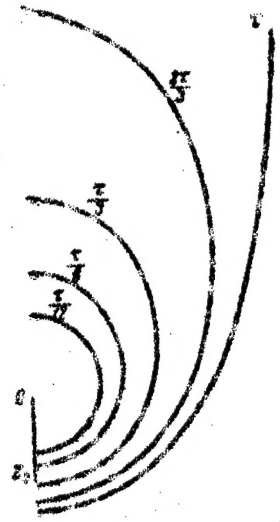

Fig. 2

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